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ISOCHRONES AND LUMINOSITY FUNCTIONS FOR OLD WHITE DWARFS

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ABSTRACT

Using a new grid of models of cooling white dwarfs, we calculate isochrones and luminosity functions in the Johnson-Kron/Cousins and HST filter sets for systems containing old white dwarfs. These new models incorporate a non-grey atmosphere which is necessary to properly describe the effects of molecular opacity at the cool temperatures of old white dwarfs. The various functions calculated and extensively tabulated and plotted are meant to be as utilitarian as possible for observers so all results are listed in quantities that observers will obtain. The tables and plots developed should eventually prove critical in interpreting the results of HST's Advanced Camera observations of the oldest white dwarfs in nearby globular clusters, in understanding the results of searches for old white dwarfs in the Galactic halo, and in determining ages for star clusters of all ages using white dwarfs. As a practical application we demonstrate the use of these results by deriving the white dwarf cooling age of the old Galactic cluster M67.

Subject headings: white dwarfs — globular clusters — dark matter – ages

1. INTRODUCTION

The search for and use of old white dwarfs in determining the ages and star formation histories in stellar systems was given an important lift recently with the publication of new sets of models of cooling white dwarfs (Hansen 1998, 1999; Saumon and Jacobson 1999). These models included non-grey atmospheres which are critical in understanding the luminosity and emergent spectrum of white dwarfs whose temperatures fall below 4000K. It is the atmosphere which regulates changes in the white dwarf's largely isothermal core and hence its cooling time. Also, the behavior of the atmosphere is strongly dependent on its composition, particularly the amount of hydrogen and helium, as helium does not form molecules whereas hydrogen does at cool temperatures. Hydrogen molecules thus provide a dramatic opacity source which must be including in the modelling in order to understand the emergent flux from the star. Thus careful teatment of the physics is essential to properly interpret the luminosities and colors of old white dwarfs.

As an added incentive, the recent microlensing results in the direction of the Magellanic Clouds (Alcock *et al.* 1997a, 1997b; Renault *et al.* 1997) suggest that a sizeable fraction (perhaps half or even larger) of the dark matter in the Galactic halo could be tied up in stellar objects with masses near $0.5M_{\odot}$. This suggests old white dwarfs as the likely candidate although other possibilities remain (neutron stars, primordial black holes or other exotica). Although all these candidates for the Galactic dark matter have their problems, numerous searches are now underway to attempt to locate these objects and there already exists a few possible old white dwarf candidates in the Hubble Deep Field (Ibata *et al.* 1999) and in the general field of the Galaxy (Harris *et al.* 1999). All the searches for old white dwarfs must be guided by appropriate cooling models, isochrones and luminosity functions. In this paper we present all the above as an aid in directing these endeavors. The compilations here are much more extensive than those in Chabrier 1999 and use a different set of models, those of Hansen. The tables and plots are all in the observers plane, in the Johnson-Kron/Cousins VRI color system or the HST system and attempts have been made to make the data as utilitarian for observers as possible. For this reason we have presented not just the cooling models but have developed white dwarf isochrones and luminosity functions both for clusters and for the Galactic halo. These are the quantities which will actually be observed when the Advanced Camera for Surveys on HST eventually penetrates to the faint end of the white dwarf cooling sequence in a globular cluster or when a wide area ground-based survey detects a sizeable sample of old halo white dwarfs.

2. THE WHITE DWARF COOLING MODELS

The white dwarf models presented here are based on the code of Hansen and Phinney 1998. The addition of new atmospheric models (Hansen 1998, 1999) has led to a revision of the cooling ages and observational appearance of old white dwarfs and it is these that we will use. The only models we present are those for C-O cores (without separation energy) and with hydrogen-rich atmospheres. Chemical separation may lengthen the ages slightly (Salaris *et al.* 1997; Hansen 1999). As such, the omission of this contribution represents the most conservative assumption, that is the fastest cooling. It is only for hydrogen-rich atmospheres that the white dwarfs remain bright (brighter than $M_V = 18$) for times comparable to the Hubble time. This is due to the strong opacity of molecular hydrogen. The helium-rich models, which do not possess this opacity source, cool much more rapidly and become fainter than $M_V = 18$ on a timescale less than 6 Gyr. Because of the strongly non-blackbody colors of cool hydrogen-rich white dwarfs caused by the hydrogen molecules redistributing the emergent flux, the stars actually become bluer in $(V - I)$ when T_{eff} drops below about 3500K. At this point there is little change in M_V as the stars evolve but the VRI colors become very different from those of black bodies *of any temperature* and these colors may be a key to the discovery of old white dwarfs.

In Figure 1 we illustrate the effect that the added H_2 opacity has on the observed cooling track of a $0.5M_\odot$ C-O core, H-rich (DA) white dwarf.

Here we compare the cooling sequence for such a model by Hansen with an extrapolated model using Wood's 1992, 1995 interiors and Bergeron *et al.'s* 1995 atmospheres. Down to an M_V of about 16 (age slightly older than 7 Gyr for both models), both sets of models agree quite well, but as the star gets cooler and more molecules are able to form, the effect of the molecular opacity increases and the two models differ enormously. *In $(V-I)$, old white dwarfs are blue not red.*

Table 1 contains these new cooling models in Johnson/Kron-Cousins VRI and Table 2 in the HST filters while Figure 2 plots the M_V, $(V-I)$ cooling sequences from Table 1. The colors can be quite different in these two different filter sets as the strong H_2 opacity produces sharp flux peaks in the emergent white dwarf spectrum.

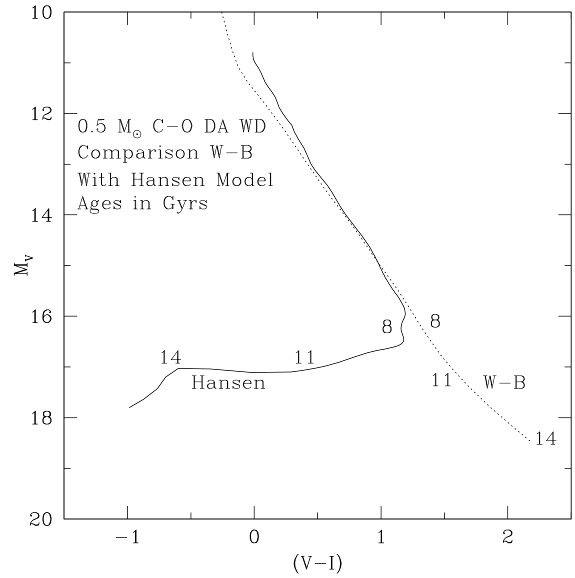

Fig. 1: New $0.5M_\odot$ white dwarf cooling model of Hansen 1998, 1999 compared with a similar mass model constructed from the interiors of Wood 1992 and Bergeron *et al.* 1995 atmospheres (W-B). The main differences set in at around 8 Gyr where the effects of atmospheric H_2 opacity become important.

The colors measured by the observer then depend critically on the positions of the transmission peaks of the filters. The HST colors are calculated using the Holtzmann *et al.* 1995 bandpasses and the transformations they use to express fluxes in V, R and I.

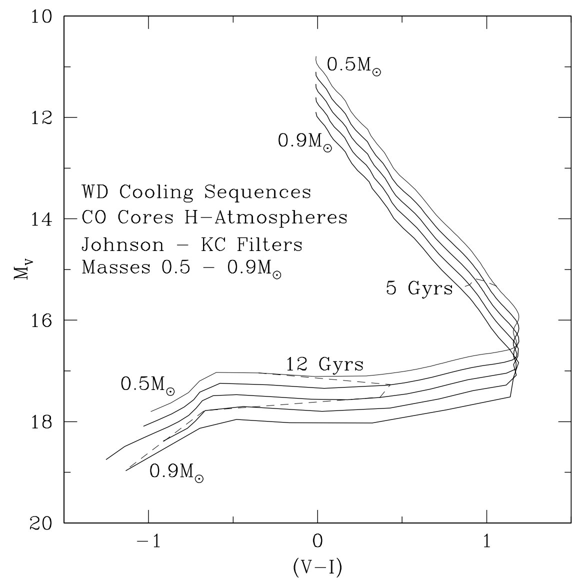

Fig. 2: Cooling sequences for C-O core, hydrogen-rich white dwarfs of varying mass. The curves shown are for Johnson-Kron/Cousins filters. Constant ages of 5 and 12 Gyr are indicated on the diagram.

The mass range in the models varies from $0.5 - 0.9M_\odot$ and the sequences all begin at about an age of 0.2 Gyr as it takes this long for the white dwarfs to lose the imprint of their initial conditions. The models terminate when T_{eff} drops below about 2000K which is the limit of the opacity tables used.

Figure 3 plots the color-color diagram in the Johnson/Kron-Cousins photometric system for a white dwarf of mass $0.7M_\odot$ with ages indicated along the sequence. The wild deviations from black body colors are evident in this diagram as the oldest and coolest white dwarfs get dramatically bluer in the $(V-I)$ color and somewhat bluer in $(V-R)$.

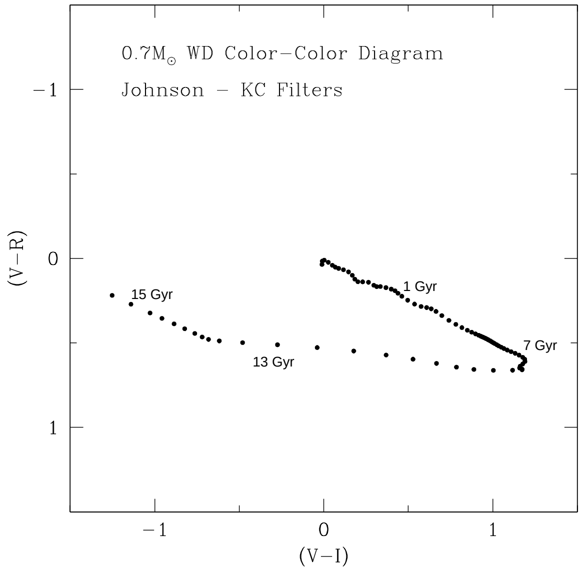

Fig. 3: $(V-R)$, $(V-I)$ color-color diagram for $0.7M_\odot$ hydrogen-rich white dwarfs in Johnson-Kron/Cousins filters. When H_2 opacity becomes important for ages older than about 8 Gyr, the colors deviate strongly from those of black bodies.

3. WHITE DWARF ISOCHRONES

When white dwarfs are observed in an open or globular cluster it is not strictly correct to compare their location in the cluster color-magnitude diagram with a theoretical cooling sequence of some mass as has generally been done in the past (e.g. Richer *et al.* 1995, 1997, 1998; Cool *et al.* 1996; Renzini *et al.* 1996). The reason for this is that the oldest white dwarfs in these clusters have evolved from the most massive stars originally in the cluster (up to the maximum mass that produces white dwarfs), and because more massive progenitors produce more massive remnants, the older white dwarfs should be more massive. This has generally not been a problem with the clusters observed thus far as the range in white dwarf masses has been relatively small, and, in any case, the initial mass function of clusters is expected to yield many fewer massive stars and hence few massive white dwarfs. However, when large ground-based telescopes or HST eventually penetrate to the termination point of the white dwarf cooling sequence in a globular cluster, and thus cover a wide range in white dwarf masses, it will be extremely important to have white dwarf isochrones ready to interpret the data as opposed to just cooling sequences for some mass.

For these reasons we have calculated isochrones for white dwarfs in star clusters with a wide range in age. All the isochrones shown are derived from solar metallicity models. The isochrones were constructed by (a) beginning with a white dwarf mass of $0.9M_\odot$, the maximum mass model that we had available, and (b) using an initial-final mass relation constructed from Herwig's 1995 data at the high mass end mated to the results from Gibson *et al.* 1999 for M67 and M4 at the low mass end to determine the mass of the main sequence progenitor. In using these data we are mixing metal rich and metal poor relations, however, at the moment this is all that can be done if we wish to stick with empirical results. (c) The stellar evolutionary models of Dominguez *et al.* 1999 were then employed to determine the lifetime of the main sequence star (A_{ms}) up to the end of the AGB. (d) The age of the white dwarf is then simply $T_{iso}-A_{ms}$ where T_{iso} is the age of the isochrone that we are calculating. (e) The absolute magnitude and color of the white dwarf of interest was then obtained from the cooling model for a white dwarf of mass $0.9M_\odot$ and age T_{iso} - A_{ms}. (f) We then decremented the mass, interpolating within the models, and repeated the process until a white dwarf mass of $0.5M_\odot$ was reached (the minimum white dwarf mass model available), at which point the calculations were halted.

Figure 4 illustrates these isochrones for a range of ages likely to be of interest in any application. The hook to the blue in the isochrones for ages less than about 7 Gyrs is not due to the effects of H_2 opacity (these stars are too hot for H_2 to form) but is caused by the fact that the white dwarfs at the bottoms of these curves come from massive main sequence stars. These produce heavier white dwarfs which follow cooling sequences which lie below those of lighter degenerates (more massive white dwarfs have smaller radii and are thus less luminous at a given temperature). It is only for ages older than 8 Gyrs where the effect of the H_2 opacity is seen.

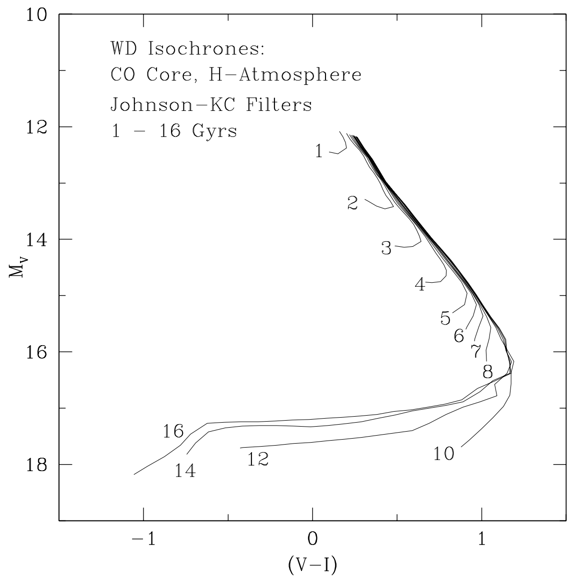

Fig. 4: White dwarf isochrones in Johnson-Kron/Cousins filters. Isochrone ages are indicated.

Figure 5 illustrates some detail for the 11 Gyr isochrone, showing the mass of the white dwarf itself and that of its main sequence precursor. Tables 3 and 4 list selected isochrones in both the Johnson-Kron/Cousins system and in the HST filters. The columns in these tables are the white dwarf mass ($M_\odot$), the mass of the progenitor ($M_\odot$) and T_{eff}, M_V, $(V-R)$ and $(V-I)$ of the white dwarf. Details of the initial-final mass relation that we used can be obtained from these Tables.

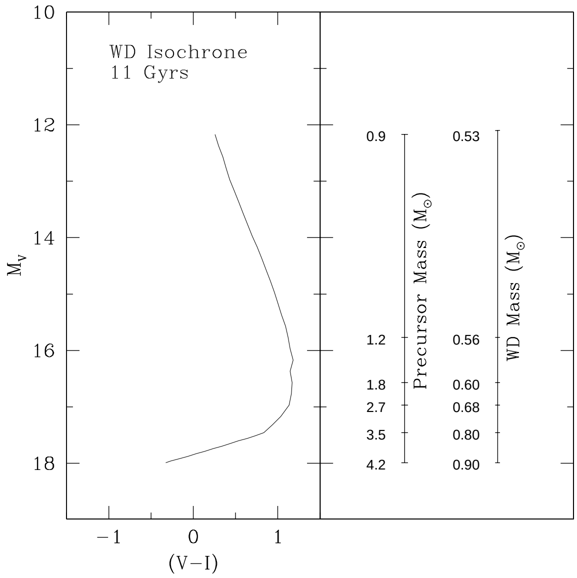

Fig. 5: Details of the 11 Gyr white dwarf isochrone indicating white dwarf and precursor masses.

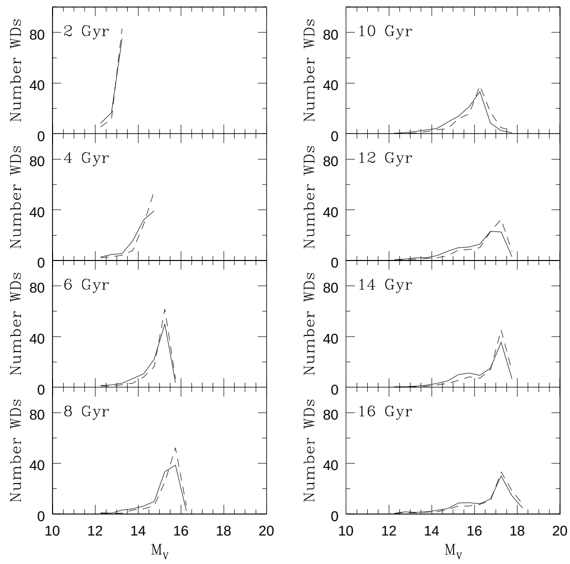

Fig. 6: Differential luminosity functions for white dwarfs in clusters of varying ages. The solid lines are for Salpeter IMFs ($\alpha = 2.35$) while the dashed lines are for significantly flatter IMFs ($\alpha = 1.3$).

4. CLUSTER WHITE DWARF LUMINOSITY FUNCTIONS

The white dwarf luminosity function in a star cluster, in the absence of dynamical evolution, contains information on the initial mass function (IMF) and age of the cluster. In fact, it will eventually be possible to use the white dwarf luminosity function in an old star cluster (e.g. a globular cluster) to extend the observed main sequence mass function up to massive stars that many billions of years ago evolved in to white dwarfs (Richer *et al.* 1997).

White dwarf luminosity functions for different ages were constructed in the following manner. (a) The isochrones and the initial-final mass relation were used to set the maximum and minimum main sequence masses for a cluster of a particular age. (b) Based on the IMF used, a random extraction of a main sequence mass in this range was made and this then yielded a white dwarf mass from the initial-final mass relation. (c) From the isochrones, the M_V of this white dwarf was then obtained. (d) This was repeated 1,000 times and eventually renormalized to 100 white dwarfs for each cluster.

Figure 6 illustrates such luminosity functions for a Salpeter IMF ($n(m) \propto m^{-\alpha}$ where $\alpha = 2.35$, solid line) and a much flatter IMF ($\alpha = 1.3$, dashed line) which is more in line with the steepest IMFs being found at the low mass end in globular clusters (Piotto and Zoccali 1999). The main feature to note in this diagram is the manner in which the peak of the cluster white dwarf luminosity function marches toward lower luminosity as the cluster age increases. This is then a potentially powerful technique for determining cluster ages that is largely independent of isochrone fitting to the turn off region of a cluster. As can be seen, the effect of even a radical change in the IMF slope has a rather small influence on the morphology of the white dwarf luminosity function and it appears that this is unlikely to be a sensitive method of investigating cluster IMFs. For this reason we only tabulate functions for Salpeter IMFs.

These luminosity functions are listed in Table 5 for those in Johnson-Kron/Cousins filters and in Table 6 for those calculated in the HST filter set. In these tables the number of white dwarfs is normalized to 100 and the columns are, respectively, the absolute V magnitude of the middle of the bin, the number of white dwarfs in that bin, the cumulative number of white dwarfs, the mean mass of the white dwarfs and of the progenitors.

As a last point regarding white dwarf luminosity functions in clusters, we inquire whether information about the age of a cluster can be obtained if the turnover in the white dwarf luminosity function is *not* observed, but only if a bright portion (e.g. to $M_V = 15$) is seen. This of course has potential practical applications as the termination points of white dwarf sequences will only be possible to observe in the nearest globular clusters even with HST and the Advanced Camera for Surveys. To investigate this we superimpose in Figure 7 synthetic white dwarf luminosity functions for 10, 12 and 14 Gyr old clusters. The numbers of white dwarfs indicated are those expected from a single WFPC2 field at 6 core radii from the center of the globular cluster M4. If the functions in Figure 7 are compared only down to $M_V = 15$, it becomes clear that virtually no useful information is obtained regarding the age of the cluster. The turnover in the luminosity function must be observed in order to constrain the cluster age.

5. THE WHITE DWARF COOLING AGE OF M67

In an earlier paper Richer *et al.* 1998 presented and discussed the observed white dwarf luminosity function in the open cluster M67 which has a turn off age of about 4 Gyr (Montgomery *et al.* 1993). Here we compare this function with synthetic luminosity functions in order to derive the white dwarf cooling age of the cluster. In the previous paper we did not have access to such synthetic functions so the current derivation of the cluster cooling age will supercede the results in the earlier paper.

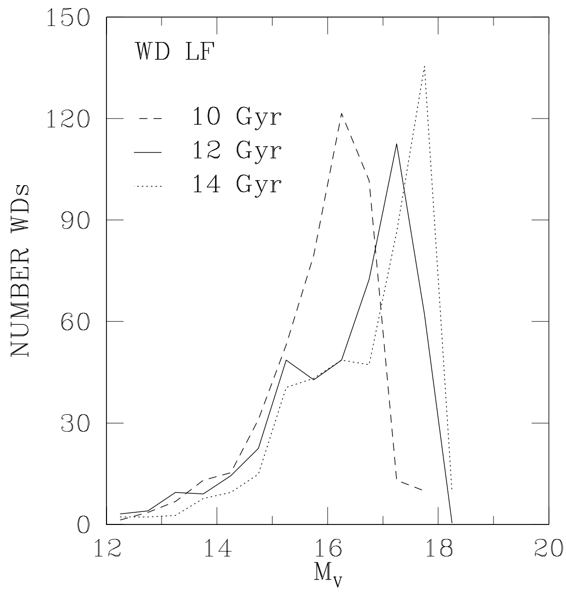

Fig. 7: Synthetic white dwarf luminosity functions in the HST filters for clusters with ages of 10, 12 and 14 Gyr. The numbers of white dwarfs are those expected in the Galactic globular cluster M4 in a single WFPC2 field at 6 core radii from the cluster center (see Richer *et al.* 1997).

Figure 8 displays the observed cumulative white dwarf luminosity function in M67 from Richer *et al.* 1998 (heavy solid line) compared with synthetic luminosity functions for clusters with ages of 3, 4 and 5 Gyrs. The synthetic

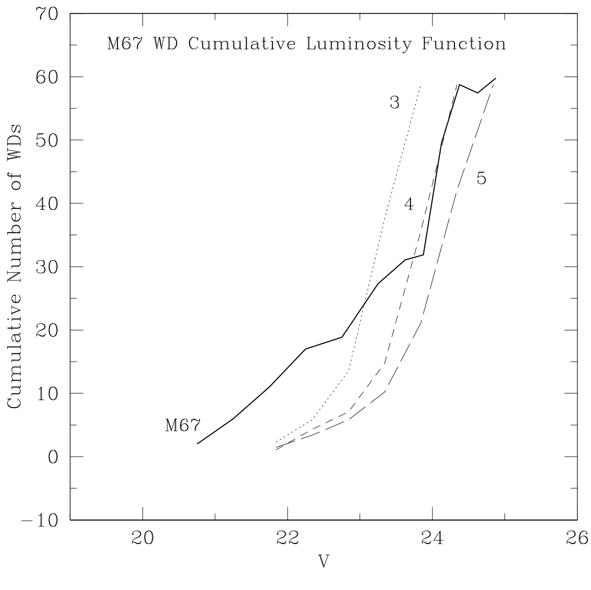

Fig. 8: The cumulative M67 white dwarf luminosity function (heavy solid line) compared with synthetic functions of ages 3, 4 and 5 Gyr, all with Salpeter IMFs. The location of the break in the M67 luminosity function and the general fit to the synthetic functions suggest a white dwarf cooling age for M67 near 4 Gyr.

functions have been shifted so as to represent a cluster with an apparent V distance modulus of 9.59 (Montgomery *et al.* 1993) and they have been normalized to contain the same number of white dwarfs that are observed in the cluster (58). All the synthetic functions have Salpeter IMFs but the actual choice of the IMF, within rather broad limits, has rather little effect on the final results as could be deduced from Figure 6. From the faint end of the luminosity function seen in M67, it is clear that the cooling age of the cluster is larger than 3 Gyr, less than 5 Gyr and that the 4 Gyr synthetic luminosity function is an excellent match to the luminosity function of the faintest observed cluster white dwarfs.

This result indicates that a properly constructed synthetic white dwarf luminosity function compared with data should be a robust and reliable age indicator, and that it will be an important tool in establishing ages for old clusters in the Galaxy.

We note in passing that the observed white dwarf luminosity function in M67 has a well populated tail of stars to high luminosity, many more stars than are predicted by the models. Varying the IMF, even by a rather large amount, could not make the fit of the synthetic function to the observations significantly better as the precursor mass range among the bright M67 white dwarfs is quite small. The origin of this tail is not currently understood but might be related to the high binary fraction in the cluster (see Richer *et al.* 1998 for further discussion) or to some deficiency in the cooling models which *overestimates* the true rate of cooling of young white dwarfs. If the binary scenario is correct, the excess number of bright white dwarfs could be produced by making a relatively large number of helium core white dwarfs via truncated stellar evolution. Such objects fade less rapidly than C-O white dwarfs as they have a greater heat capacity per unit mass.

6. THE WHITE DWARF LUMINOSITY FUNCTIONS IN THE GALACTIC HALO

The microlensing experiments in the direction of the LMC seem to be indicating that $60 \pm 20\%$ of the dark matter in the Galactic halo is tied up in $0.5^{+0.3}_{-0.2}M_\odot$ objects (Alcock *et al.* 1997a, 1997b; Renault *et al.* 1997). This naturally suggests old white dwarfs although other possibilities exist (e.g. neutron stars, primordial black holes). The possibility that white dwarfs are important contributors to the Galactic dark matter has been considered for some time now (Larson 1986; Silk 1991; Carr 1994), but with the microlensing results this scenario has taken on increased viability.

Chabrier 1999, Chabrier *et al.* 1996, Gibson and Mould 1997, and Adams and Laughlin 1996 have all pointed out that if indeed this scenario is correct, the IMF of the white dwarf precursors could not have had a Salpeter form but might have been more Gaussian in shape and peaked near $2.7M_\odot$. For this reason we have calculated halo luminosity functions for both Salpeter IMFs and those of the form $\Phi(m) = \exp^{-(\overline{m}/m)^{\beta_1}}\, m^{-\beta_2}$ with $\overline{m} = 2.7$, $\beta_1 = 2.2$ and $\beta_2 = 5.75$ (Chabrier *et al.* 1996).

Under this scenario, old white dwarfs will be plentiful in the Galactic halo but difficult to detect because of their intrinsic faintness. The local number of such objects can be determined simply from the local dark matter density ($0.0079M_\odot/\mathrm{pc}^3$) (Alcock *et al.* 1997a; Chabrier and Méra 1997; Gould, Flynn and Bahcall 1996) and the mean white dwarf mass ($\langle M_{WD}\rangle$) through

$$Local\ Number\ WDs\ /pc^3 = \frac{0.0079}{\langle M_{wd}\rangle}.$$

A synthetic halo white dwarf luminosity function for a given age (T_{field}) and limiting magnitude (V_{lim}) was calculated as follows. First, from the cluster luminosity function for an age T_{field}, we obtained the maximum and minimum main sequence masses as before as well as the brightest white dwarf (M_V^{min}) in the cluster. We then set the distance R_{max} out to which we would fill a volume with white dwarfs as

$$R_{max} = 10^{\frac{(V_{lim} - M_V^{min} + 5)}{5}}.$$

From the IMF we then extracted a main sequence star of a given mass and determined the associated white dwarf mass M_{wd} through the initial-final mass relation. This white dwarf was then placed randomly at a distance of R_{wd} inside the volume so as to keep the density constant. From the isochrone we then obtained the M_V of this object and obtained its apparent V magnitude from

$$V = M_V - 5 + 5\ log\ R_{wd}.$$

The average mass of the white dwarfs in the volume came simply from

$$\langle M_{wd} \rangle = \sum \frac{M_{wd}}{N},$$

and the total number of white dwarfs observed in the volume would then be

$$N_{tot} = \frac{0.0079}{\langle M_{wd} \rangle} * \frac{4}{3}\pi R_{max}^3.$$

This was done N times until $N \geq N_{tot}$ at which point the calculations were terminated.

In this way we constructed synthetic halo luminosity functions including all the stars in the mass range allowed by the models for ages of 10, 14 and 16 Gyrs. The volume size was chosen so that all the stars brighter than $V = 28$ would be counted. This limiting magnitude constitutes a reasonably faint limit but not so faint that the halo density variation would be important for luminosity functions constructed with Chabrier *et al.* 1996 IMFs. With $V_{lim} = 28$ and $M_V^{min} = 14.5$, 15.5 and 16.0 for ages of 10, 14 and 16 Gyr with a Chabrier IMF, R_{max} was 5.0, 3.2 and 2.5 kpc respectively. However, when we used a Salpeter IMF it was 12.5 kpc for all ages. For the Chabrier IMFs the number of stars inserted in to the volume was $\sim 1.6 \times 10^5$ per square degree of field for the 10 Gyr halo ($\sim 2.0 \times 10^4$ for 16 Gyr) while it was $\sim 2.5 \times 10^6$ for the Salpeter IMF at all 3 ages.

We can only reasonably calculate halo luminosity functions for stars that are very local (so as to keep the density constant and not to have the functions depend on the direction of viewing) so this is a reasonable assumption for Chabrier-type IMFs (about a 6% error is made by not including the R^{-2} halo density variation for a 16 Gyr halo for viewing towards the Galactic poles), but not very good for Salpeter functions (looking in the same direction, an overestimate of about a factor of 4 in the counts from the most distant region of the volume results from ignoring the density fall off). This is not a critical point as, if indeed the MACHOs are old white dwarfs, their precursors are unlikely to have been formed with a Salpeter IMF. In any case, the Salpeter IMF counts exceed those with Chabrier IMFs by a much larger amount than that caused by density variations. For example, for a 14 Gyr halo and a one square degree field to $V = 27.5$, the cumulative counts with HST filters for a Salpeter IMF are 34,000 (assuming constant density) to be compared with 1,210 for those with a Chabrier IMF. The average white dwarf mass for a halo of this age with a Chabrier IMF was $0.62 M_\odot$ while it was $0.57 M_\odot$ for a Salpeter function.

Tables 7 and 8 contain halo white dwarf luminosity functions for Johnson-Kron/Cousins and HST filter systems, respectively, for ages of 10, 14 and 16 Gyr all with Chabrier IMFs. For reasons outlined above, the functions for Salpeter IMFs are not very accurate so they should be considered as illustrative only. Even so, there are a number of interesting conclusions that can be derived from the plots (Figure 9) and Tables 7 and 8. First, the white dwarf luminosity functions based on a Salpeter IMF always predict many more stars than do those for a Chabrier function. For example, the Salpeter-based function for 14 Gyr predicts that 5 old white dwarfs should be found in the Hubble Deep Field (HDF) to $V = 26.0$ whereas there is at most 1 possible candidate (Ibata *et al.* 1999, Hansen 1999). This by itself seems capable of excluding old white dwarfs formed with a Salpeter IMF as the entire source of the Galactic dark matter. By contrast, using the luminosity function generated with a Chabrier IMF suggests that only 0.2 should be found. This model is clearly not as yet excluded by the data.

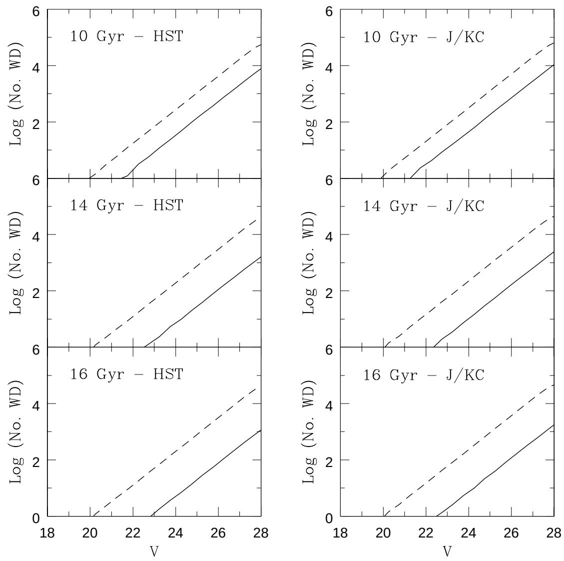

Fig. 9: Cumulative white dwarf luminosity functions in the Galactic halo. The y-axis is the logarithm of the number of stars per square degree under the assumption that white dwarfs make up 100% of the local dark matter density and that they all have hydrogen atmospheres. Solid lines are for a Chabrier 1999 type IMF while the dashed line is for a Salpeter IMF.

For a 10 Gyr old halo and a Chabrier IMF, to $V = 28.0$, we expect to find 14 old white dwarfs in the HDF (if the dark halo is 100% hydrogen-rich white dwarfs) but only 2 if it is as old as 16 Gyr. This, then, coupled with the colors of the stars, is a potentially powerful method of establishing both the time of formation of the Galactic halo and possibly shedding some light on the nature of dark matter in the Galaxy. With the advent of the Advanced Camera for Surveys on HST, experiments of this sort covering larger areas than the HDF will become eminently feasible.

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